

Brane Cosmology for Vacuum and Cosmological Constant Bulk

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Abstract

We consider the cosmology of a 3-brane universe, in a five dimensional space time (Bulk). We present some solutions to the five-dimensional Einstein equation, where a perfect fluid is confined to the 3-brane. We investigate the evolution of brane for two models of bulk. We choose ordinary and modified polytropic gas in brane and it is seen that these models have some new features.

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1 Introductions

Today, cosmology is an active field on physics. Its main goal is to describe the evolution of our universe from initial time to its present form. The mathematical description of cosmology is provided by the Einstein equations. There has been a lot of activity in the domain of cosmology with extra dimension. In the string theory the Einstein equations are part of the theory but the theory itself is consistent only in higher than four dimension. It has been suggested that [1], our four-dimensional universe would be a 3-brane living in a $(4+d)$ -dimensional spacetime where d is the extra dimension. A strong motivation of considering such models comes from strongly coupled string theory [19]. Indeed, Randall and Sundrum [2],[3], Geobersashvili [4] for $d = 1$, (reviving the idea of [5]), have shown that the extra dimension need not even be compact. This result is in contrast with the main investigation on the possibility of extra dimension which began with Kaluza-Klein type theory (see e.g [6]). The theory of Kaluza-Klein explain that the extra dimensions are compactified on a small enough radius to evade detection. Randall and Sundrum have shown that these kind of models give back to standard four-dimension gravity [7],[8],[9]. For studying the cosmological evolution of this model, somebody has assumed that the density is localized on the brane, while the bulk energy momentum tensor is only a negative cosmological constant [7]. Some modification of the cosmological evolution which is due to the adding interaction in the gravitation sector, such as Gauss-Banet term in the bulk [10] and an induced gravity term on the brane [11]. Another modifications have considered for the case which the bulk contains some matter component in addition to the negative cosmological constant [12],[13],[14]. The cosmological evolution is studied for two system A and B. In system A, brane located at a fixed value of the fourth spatial coordinate, and the bulk is time-depended. And in the system B, the bulk is static and the brane is moving. In the case of a vacuum bulk, the brane evolution is discussed in two system of coordinate A [7],[8] and B [9],[17].

In the present work, our investigation is to solve the Einstein equation in the brane world scenario for a perfect fluid called polytropic gas on the brane for system A. It is well known that in standard cosmology a special kind of fluid is called polytropic gas [18]. This kind of fluid has some very interesting properties, and hence, in this paper we discussed the evolution of a homogeneous isotropic brane world filled with a polytropic and modified polytropic gas, when the bulk contain either vacuum or a cosmological constant.

The paper is organized as follow: In section 2, we have introduced some

basic equation that are useful for our investigation. In section 3, assuming a equation of state of polytropic gas for brane, then we have obtained energy density of brane with respect to scale factor of brane. In subsection 3.1, we have supposed a vacuum bulk, and it is shown that we have a deceleration universe for early time. In late time, it is shown that, we have a universe with $\ddot{a}_0 = 0$. In subsection 3.2, we have assumed a cosmological constant bulk. So, it is shown that in early time we have similar condition as 3.1, but in late time, we can have an accelerate or decelerate universe. In section 4, we have chosen a modified polytropic for brane and obtained energy density of brane with respect to scale factor of brane. We consider two case for α , and it is shown that the equation of state for $-1 < \alpha < 0$ is not a suitable case. Another case is $\alpha = 1 + \frac{1}{n}$. In subsection 4.1, we have a vacuum bulk for this kind of brane, and for early time we arrive at a decelerate universe; but for late time in contrary with the first section, we have an accelerate universe. In subsection 4.2, we have got a cosmological constant bulk model. It is shown that for early time the condition are similar to subsection 4.1, and we show that the cosmological constant, for this kind of brane, must be positive.

2 Preliminary

We suppose that the geometry of the five-dimensional bulk is characterized by a spacetime metric as:

$$ds^2 = -n^2(t, y)dt^2 + a^2(t, y)\delta_{ij}dx^i dx^j + b^2(t, y)dy^2, \quad (1)$$

where y is the fifth coordinate and δ_{ij} is a maximally symmetric 3-dimensional metric. The Einstein equations in 5-dimensional is as follow:

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = \kappa^2 T_{\alpha\beta}, \quad (2)$$

where $R = g^{\alpha\beta}R_{\alpha\beta}$, $R_{\alpha\beta}$ is the 5-dimensional Ricci-tensor κ is related to the 5-dimensional Newton's constant, $G_{(5)}$, and the 5-dimensional reduced Plank mass $M_{(5)}$ is as:

$$\kappa^2 = 8\pi G_{(5)} = M_{(5)}^{-3}. \quad (3)$$

Now, for obtaining the solution of (2), we must specify the matter really. We classify the matter in two distinct categories, i)matter confined to our brane univers (3-brane), ii)matter distributed over the 5-dimensional bulk. So that, the energy momentum tensor take the form:

$$T^\alpha_\beta = T^\alpha_\beta|_{bu} + T^\alpha_\beta|_{br}, \quad (4)$$

Where $T^\alpha_\beta|_{bu}$ is the energy momentum of the bulk matter, and an explicit form of that is

$$T^\alpha_\beta|_{bu} = \text{diag}(-\rho_{bu}(t, y), p_{bu}, p_{bu}, p_{bu}, p_{5bu}), \quad (5)$$

the second term $T^\alpha_\beta|_{br}$ is related to the matter content in the brane ($y = 0$), whereas we consider homogeneous and isotropic geometries inside the brane, one can write the energy-momentum tensor in the brane as:

$$T^\alpha_\beta|_{br} = \frac{\delta(y)}{b} \text{diag}(-\rho_{br}, p_{br}, p_{br}, p_{br}, 0). \quad (6)$$

So this assumption, clearly show that energy density, ρ_{br} , and pressure, p_{br} , are independent of the position inside the brane. It is seen that $T_{05} = 0$, it means that there is no flow of matter along the fifth dimension. Therefore, using Eq.(1) and Eq.(2), one can arrive at

$$G_{00} = 3 \left\{ \frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left(\frac{a''}{a} + \frac{a'}{a} \left(\frac{a'}{a} - \frac{b'}{b} \right) \right) + k \frac{n^2}{a^2} \right\}, \quad (7)$$

$$G_{ij} = \frac{a^2}{b^2} \gamma_{ij} \left\{ \frac{a'}{a} \left(\frac{a'}{a} + 2 \frac{n'}{n} \right) - \frac{b'}{b} \left(\frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right\} \\ + \frac{a^2}{n^2} \gamma_{ij} \left\{ \frac{\dot{a}}{a} \left(-\frac{\dot{a}}{a} + 2 \frac{\dot{n}}{n} \right) - 2 \frac{\ddot{a}}{a} + \frac{\dot{b}}{b} \left(-2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right), -\frac{\ddot{b}}{b} \right\} - k \gamma_{ij} \quad (8)$$

$$G_{05} = 3 \left(\frac{n'}{n} \frac{\dot{a}}{a} + \frac{a'}{a} \frac{\dot{b}}{b} - \frac{\dot{a}'}{a} \right), \quad (9)$$

$$G_{55} = 3 \left\{ \frac{a'}{a} \left(\frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left(\frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \right) - k \frac{b^2}{a^2} \right\}, \quad (10)$$

where $\dot{\chi} = \frac{d\chi}{dt}$ and $\chi' = \frac{d\chi}{dy}$. In order to have a well defined geometry, the metric tensor must be continuous across the brane (i.e $y = 0$) while its derivative with respect to y can be discontinuous on the brane. As a result, the Dirac delta function will appear in the second-order derivative of metric coefficient with respect to y . So according to [7], we can obtain

$$\frac{[a']}{b_0 a_0} = -\frac{\kappa^2}{3} \rho_{br}, \quad (11)$$

$$\frac{[n']}{b_0 n_0} = \frac{\kappa^2}{3} (3p_{br} + 2\rho_{br}), \quad (12)$$

where the subscript 0 in the scale factor indicate their values on the brane. We take the jump of Eq.(9) and using Eqs.(11,12), we can obtain

$$\dot{\rho}_{br} + 3\frac{\dot{a}_0}{a_0}(\rho_{br} + p_{br}) = 0, \quad (13)$$

then, the average value of Eq.(10) for $y \rightarrow 0^+$ and $y \rightarrow 0^-$ and impose the Z_2 -symmetry, we arrive at [7]

$$\frac{\ddot{a}_0}{a_0} + \frac{\dot{a}_0^2}{a_0^2} = -\frac{\kappa^4}{36}\rho_{br}(\rho_{br} + 3p_{br}) - \frac{\kappa^2}{3b_0^2}p_{5bu} - \frac{k}{a_0^2} \quad (14)$$

By using a suitable time transformation, we can choose $n_0 = 1$. Eq.(14) is known as generalize Friedmann type equation on the brane. It has a basic difference from the ordinary Friedmann equation of standard cosmology. In fact, Eq.(14) H depends quadratically on the brane energy density, in contract with the usual linear dependence in a ordinary Friedmann equation. We can obtain the 5D-field equations (7) and (10) in the compact form as [7]:

$$\psi' = -\frac{2}{3}a'a^3\kappa^2\rho_{bu}, \quad (15)$$

$$\dot{\psi} = \frac{2}{3}\dot{a}a^3\kappa^2p_{5bu}, \quad (16)$$

where

$$\psi \equiv \frac{(a'a)^2}{b^2} - \frac{(\dot{a}a)^2}{n^2} - ka^2. \quad (17)$$

By equating the y-derivative of Eq.(16) with the time-derivative of Eq.(15), we have

$$a'\rho_{bu} + \dot{a}p'_{5bu} + (\rho_{bu} + p_{5bu})\left(\dot{a}' + 3\frac{\dot{a}a'}{a}\right) = 0, \quad (18)$$

and, the constraint $\nabla_\alpha G^{\alpha 0} = 0$, gives

$$\rho_{bu} + 3\frac{\dot{a}}{a}(\rho_{bu} + p_{bu}) + \frac{\dot{b}}{b}(\rho_{bu} + p_{5bu}) = 0. \quad (19)$$

Using the results of junction condition (11) and imposing the Z_2 -symmetry and also taking an mean value of equation (17) for $y \rightarrow 0^+$ and $y \rightarrow 0^-$, one can obtain the generalize Friedmann equaton on the brane as

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^4}{36}\rho_{br}^2 - \frac{\psi_0(t)}{a_0^4} - \frac{k}{a_0^2}. \quad (20)$$

3 Polytropic Brane Model

We now consider a polytrop gas model of DE [18] which satisfies the equation of state

$$p_{br} = A\rho_{br}^{1+\frac{1}{n}}, \quad (21)$$

where A is a constant and n is the polytropic index[19]. From the energy conservation equation (13), the expression for matter density is given by

$$\rho_{br} = \left(\frac{1}{\rho_1 a_0^{\frac{3}{n}} - A} \right)^n, \quad (22)$$

where ρ_1 is an arbitrary integration constant. We clearly see that if A be negative, a_0 can lead to zero. But if A be positive, a_0 can not lead to zero, because with pass of time from zero, the denominator of Eq.(22) decrease and ρ_{br} increase. Then in special time the energy density goes to infinite. It means with pass of time, energy density increase and physically it is not true. So for positive value of A the model does not have a singularity, and we have a minimum value for brane scale factor in early time i.e a_0 should be larger than $\left(\frac{A}{\rho_1} \right)^{\frac{n}{3}}$.

3.1 Vacuum Bulk With Polytropic Gas in Brane

Whereas there is no matter in the Bulk, we have $T^\alpha_\beta|_{bu} = 0$. Therefore, Eqs.(15) and (16) show that $\psi = D$ should be constant. Then, the generalized Friedmann equation is as

$$\left(\frac{\dot{a}_0}{a_0} \right)^2 = \frac{\kappa^4}{36} \rho_{br}^2 - \frac{D}{a_0^4} - \frac{k}{a_0^2}, \quad (23)$$

and from Eq.(14) we have

$$\frac{\ddot{a}_0}{a_0} + \frac{\dot{a}_0^2}{a_0^2} = -\frac{\kappa^4}{36} \rho_{br}(\rho_{br} + 3p_{br}) - \frac{k}{a_0^2}. \quad (24)$$

By using these results we arrive at:

$$\dot{a}_0^2 = \frac{\kappa^4}{36} \left(\frac{1}{\rho_1 a_0^{\frac{3}{n}} - A} \right)^{2n} a_0^2 - \frac{D}{a_0^2} - k, \quad (25)$$

and

$$\ddot{a}_0 = -\frac{\kappa^4}{18} \left(\frac{1}{\rho_1 a_0^{\frac{n}{3}} - A} \right)^{2n} a_0 - \frac{\kappa^4}{12} A \left(\frac{1}{\rho_1 a_0^{\frac{n}{3}} - A} \right)^{2n+1} a_0 + \frac{D}{a_0^3}. \quad (26)$$

Here, we continue our investigation for various value of A :

- **For $A < 0$**

From (25) and (26), we realize when $a_0 \rightarrow 0$, \dot{a}_0^2 lead to $-\frac{D}{a_0^2}$. Also \dot{a}_0^2 must be positive, as a result

$$D < 0. \quad (27)$$

In this era $\ddot{a}_0 \rightarrow \frac{D}{a_0^3}$. Because D is a negative constant, we have a decelerate universe in early time.

In late time (i.e $a_0 \rightarrow \infty$), $\dot{a}_0^2 \rightarrow -k$ and $\ddot{a}_0 \rightarrow 0$, too. Also, k can be 0 or -1 .

- **For $A > 0$**

In this case for investigation of universe in early time we should lead a_0 toward to $\left(\frac{A}{\rho_1}\right)^{\frac{3}{n}}$. So, as a result $\dot{a}_0^2 \rightarrow +\infty$ and $\ddot{a}_0 \rightarrow -\infty$. It shows that we have a decelerate universe. For late time when we lead a_0 toward to infinity, $\dot{a}_0^2 \rightarrow -k$ and $\ddot{a}_0 \rightarrow 0$; like to the previous case.

3.2 Cosmological Constant Bulk With Polytropic Gas in Brane

In this case, we choose equation of state as $p_{bu} = p_{5bu} = -\rho_{bu}$ in the bulk. From conservation relation (18) and (19) we arrive at this result that, $\rho_{bu} = \Lambda$ must be a constant. According to Eqs.(15,16), we have

$$\psi(t, y) = -\frac{\kappa^2}{6} \Lambda a^4 + D, \quad (28)$$

D is a integration constant. Now, with substituting this result in evolution equations of brane, we obtain

$$\dot{a}_0^2 = \frac{\kappa^4}{36} \left(\frac{1}{\rho_1 a_0^{\frac{n}{3}} - A} \right)^{2n} a_0^2 + \frac{\kappa^2}{6} \Lambda a_0^2 - \frac{D}{a_0^2} - k, \quad (29)$$

and

$$\begin{aligned}\ddot{a}_0 = & -\frac{\kappa^4}{18} \left(\frac{1}{\rho_1 a_0^{\frac{3}{n}} - A} \right)^{2n} a_0 - \frac{\kappa^2}{12} A \left(\frac{1}{\rho_1 a_0^{\frac{3}{n}} - A} \right)^{2n+1} a_0 \\ & - \frac{\kappa^2}{3} \left(\frac{1}{2} - \frac{1}{b_0^2} \right) \Lambda a_0 + \frac{D}{a_0^3}.\end{aligned}\quad (30)$$

In the follow, we investigate evolution of universe for negative and positive values of A :

- **For $A < 0$:**

In this case, we can lead a_0 toward to zero, so when $a_0 \rightarrow 0$, $\dot{a}_0 \rightarrow -\frac{D}{a_0^2}$. For having a positive \dot{a}_0^2 , D must be negative; namely

$$D < 0. \quad (31)$$

Also, in this time, $\ddot{a}_0 \rightarrow \frac{D}{a_0^3}$. This situation is like to the previous subsection, and we again have a decelerate universe in early time.

In late time (when $a_0 \rightarrow \infty$), $\dot{a}_0^2 \rightarrow \frac{\kappa^2}{6} \Lambda a_0^2$. Therefore, because \dot{a}_0^2 is positive, so Λ must be positive ($\Lambda > 0$). Also, in this era $\ddot{a}_0 \rightarrow -\frac{\kappa^2}{3} \left(\frac{1}{2} - \frac{1}{b_0^2} \right) \Lambda a_0$. It means if $b_0^2 > 2$, we have a deceleration and if $b_0^2 < 2$ we have an acceleration. Note that if $b_0^2 = 2$, \ddot{a}_0 is equal to zero, and we do not have any deceleration or acceleration universe.

- **For $A > 0$:**

In this case, a_0 tends to $\left(\frac{A}{\rho_1} \right)^{\frac{n}{3}}$ for investigation of universe on early time. As $a_0 \rightarrow \left(\frac{A}{\rho_1} \right)^{\frac{n}{3}}$; $\dot{a}_0^2 \rightarrow +\infty$ and $\ddot{a}_0 \rightarrow -\infty$. So, it is shown that we have decelerate universe. For late time, when $a_0 \rightarrow \infty$, $\dot{a}_0^2 \rightarrow \frac{\kappa^2}{6} \Lambda a_0^2$. It is clearly seen that, is Λ be positive \dot{a}_0^2 will become a positive quantity. Also $\ddot{a}_0 \rightarrow -\frac{\kappa^2}{3} \left(\frac{1}{2} - \frac{1}{b_0^2} \right) \Lambda a_0$, because of $\Lambda > 0$, we have a decelerate universe for $b_0^2 > 2$, and an accelerate universe for $b_0^2 < 2$. And if $b_0^2 = 2$ there is not any deceleration or acceleration universe.

4 Modified Polytrropic Brane Model

In this section we get another equation of state for brane as:

$$p_{br} = -\rho_{br} + A\rho_{br}^\alpha, \quad (32)$$

where A and α are constant. With the help of brane conservation equation, Eq.(13), one can obtain

$$\rho_{br}^{1-\alpha} = \ln \left(\frac{1}{\rho_1 a_0^{3A(1-\alpha)}} \right). \quad (33)$$

ρ_1 is a integration constant. IN the follow, we discuss about two case for α .

- **If $-1 \leq \alpha < 0$.**

In this case equation (32) shows the equation of state of modified chaplygin gas. According to Eq.(33), we arrive at

$$\rho_{br} = \left(-3A(1-\alpha) \ln(\rho_1' a_0) \right)^{\frac{1}{1-\alpha}}. \quad (34)$$

If A be a positive constant ρ_{br} does not become a physical. So, we take A as a negative constant. With a little attention, we realized that, a_0 can not be smaller than $\left(\frac{1}{\rho_1'}\right)$. It means we have a minimum value for a_0 in initial time. Here, it is clearly seen that the energy density tends to infinite for late time, and this is incompatible with experiment. Then, this case is not a suitable equation of state; also, for $\alpha < 1$ we have this situation. So, the equation of state which are combination of cosmological constant and every term of $A\rho^\alpha$ (where $\alpha < 1$) are not a good equation of state. For instance, the equation of state of chaplygin gas and generalized chaplygin gas can not be with cosmological constant considered.

- **If $\alpha = 1 + \frac{1}{n}$.**

In this case equation (32) shows the equation of state of modified polytropic gas, also in here we take A as a positive constant. So, from (33) we have

$$\rho_{br} = \left(\frac{n}{3A \ln(\rho_1' a_0)} \right)^n. \quad (35)$$

From this relation, it is clearly seen that a_0 can not be smaller than $\left(\frac{1}{\rho_1'}\right)$, so at initial time we have a minimum size for a_0 , and in that time a_0 should be larger than $\left(\frac{1}{\rho_1'}\right)$. Then for investigation of universe in early time a_0 tends to one.

At the continuance we investigate the evolution of universe for two model of bulk.

4.1 Vacuum Bulk With Modified Polytrropic Gas in Brane

Here we assume vacuum for bulk. Because we have no matter in the bulk, we can write $T^\alpha_\beta|_{bu} = 0$. So according to (15) and (16), $\psi = D$ is constant. Now, we obtain the evolution of universe as follow. Substituting (35) in (14) and (20),

it is easily obtained

$$\dot{a}_0^2 = \frac{\kappa^4}{36} \left(\frac{n}{3A \ln(\rho'_1 a_0)} \right)^{2n} a_0^2 - \frac{D}{a_0^2} - k, \quad (36)$$

also

$$\begin{aligned} \ddot{a}_0 &= \frac{\kappa^4}{36} \left(\frac{n}{3A \ln(\rho'_1 a_0)} \right)^{2n} a_0 - \frac{\kappa^4}{12} A \left(\frac{n}{3A \ln(\rho'_1 a_0)} \right)^{2n+1} a_0 \\ &\quad + \frac{D}{a_0^3}. \end{aligned} \quad (37)$$

From (36) and (37), we consider the state of our universe. In early time a_0 tends to $\left(\frac{1}{\rho'_1}\right)$, so $\dot{a}_0^2 \rightarrow +\infty$ and $\ddot{a}_0 \rightarrow -\infty$

$$\ddot{a}_0 \rightarrow \frac{\kappa^4}{12} \left(\frac{n}{3A \ln(\rho'_1 a_0)} \right)^{2n} a_0 \left(\frac{1}{3} - \frac{n}{3 \ln(\rho'_1 a_0)} \right)$$

; it means that, we have a decelerate universe in early time.

In late time when a_0 tends to infinite, $\dot{a}_0^2 \rightarrow +\infty$, also $\ddot{a}_0 \rightarrow +\infty$. So, we have an accelerate universe in late time.

4.2 Cosmological Constant Bulk With Modified Polytrropic Gas in Brane

In this case, we impose $p_{bu} = p_{5bu} = -\rho_{bu}$ as equation of state of bulk. Using (18) and (19) we arrive to this result that $\rho_{bu} = \Lambda$ is constant. Eqs.(15) and (16) and this result help us to write ψ as:

$$\psi(t, y) = -\frac{\kappa^2}{6} \Lambda a^4 + D. \quad (38)$$

In the continuance, like to the previous subsection we investigate the evolution of universe for some specific value of α .

The evolution equations are as

$$\dot{a}_0^2 = \frac{\kappa^4}{36} \left(\frac{n}{3A \ln(\rho'_1 a_0)} \right)^{2n} a_0^2 + \frac{\kappa^2}{6} \Lambda a_0^2 - \frac{D}{a_0^2} - k, \quad (39)$$

and

$$\begin{aligned} \ddot{a}_0 = & \frac{\kappa^4}{36} \left(\frac{n}{3A \ln(\rho'_1 a_0)} \right)^{2n} a_0 - \frac{\kappa^4}{12} A \left(\frac{n}{3A \ln(\rho'_1 a_0)} \right)^{2n+1} a_0 \\ & + \frac{\kappa^2}{6} \Lambda a_0 + \frac{D}{a_0^3}. \end{aligned} \quad (40)$$

From (39) and (40), as $a_0 \rightarrow \left(\frac{1}{\rho'_1}\right)$, then $\dot{a}_0^2 \rightarrow +\infty$ and $\ddot{a}_0 \rightarrow -\infty$. It means we have a decelerate universe in early time.

As $a_0 \rightarrow \infty$, $\dot{a}_0^2 \rightarrow \frac{\kappa^2}{6} \Lambda a_0^2$, then Λ must be positive to give a positive value for \dot{a}_0^2 . Also, $\ddot{a}_0 \rightarrow \frac{\kappa^2}{6} \Lambda a_0$; namely, we have an accelerate universe in late time.

5 conclusion

It has shown that for polytropic model of barne with $A < 0$, we have a limited value for brane energy density at initial time, and for $A > 0$ we have a nonzero value for brane scale factor. It is shown that, for vacuum bulk model we have a decelerate universe; Also in late time for both positive and negative value of A , we have $\ddot{a}_0 = 0$. In the cosmological bulk model, the condition of universe in early time is like to the vacuum bulk model; however, in late time dependence on the value of b_0^2 , we can have an accelerate universe.

In the second model of brane we choose two case for α . For $-1 \leq \alpha < 0$, it is shown that the constant in equation of state should be negative, also it is realized that this case is not a suitable model. When $\alpha = 1 + \frac{1}{n}$, it has shown that a_0 in initial time has a minimum value. we find that for two model of bulk there is a decelerate universe in early time and an accelerate universe in late time; also we arrive at this result that, the cosmological constant of bulk, in this model, must be positive.

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